Arithmetic Sequence

Lesson Vocabulary
- arithmetic sequence
- common difference
- arithmetic mean

Objective To define, identify, and apply arithmetic sequences

An **arithmetic sequence**, also known as a *discrete linear function*, is a sequence for which consecutive terms have a *common difference*. For instance, the terms of the sequence 0, 7.5, 15, 22.5, 30, 37.5, 45, 52.5, 60, 67.5, 75, 82.5, 90, 97.5, 105, have a common difference of 7.5.

The **arithmetic mean**, or average, of two numbers x and y is $\frac{x + y}{2}$.

take note

Key Concept Arithmetic Sequence

An arithmetic sequence with a starting value a and common difference d is a sequence of the form

$$a, a + d, a + 2d, a + 3d, \ldots .$$

A recursive definition for this sequence has two parts:

$a_1 = a$ initial condition

$a_n = a_{n-1} + d$, for $n > 1$ recursive formula

An explicit definition for this sequence is a single formula:

$a_n = a + (n - 1)d$, for $n \geq 1$

Essential Understanding In an *arithmetic sequence*, the difference between any two consecutive terms is always the same number. You can build an arithmetic sequence by adding the same number to each term.

Arithmetic Sequence

In general, when 0 is the starting position for the initial term a of an arithmetic sequence

the sequence has the explicit rule $f(n) = a + dn$ for $n \geq 0$

and the recursive rule $f(0) = a$

and $f(n) = f(n-1) + d$ for $n \geq 1$.

n	0	1	2	3	4	5
$f(n)$	−7	−2	3	8	13	18

When 1 is the starting position of the initial term, the sequence has the

explicit rule $f(n) = a + d(n-1)$ for $n \geq 1$

n	1	2	3	4	5	6
$f(n)$	11	5	−1	−7	−13	−19

and the recursive rule $f(1) = a$

and $f(n) = f(n-1) + d$ for $n \geq 2$.

Arithmetic Sequence

Identifying Arithmetic Sequences

Is the sequence an arithmetic sequence?

A **3, 6, 9, 12, 15, . . .**

Find the differences between consecutive terms.

Each difference is 3. The sequence has a common difference. The sequence is an arithmetic sequence.

B **1, 4, 9, 16, 25, . . .**

Find the difference between consecutive terms.

There is no common difference. The sequence is not an arithmetic sequence.

Arithmetic Sequence

Identifying Arithmetic Sequences

1. Is the sequence an arithmetic sequence?

a. 2, 4, 8, 16, . . .

b. 1, 5, 9, 13, 17, . . .

Arithmetic Sequence

Analyzing Arithmetic Sequences

A **What is the 100th term of the arithmetic sequence that begins 6, 11, . . .?**

The first term a is 6. The common difference d is $11 - 6 = 5$.

$a_n = a + (n - 1)d$ Use the explicit formula.

$a_n = a + (n - 1)d$ Use the explicit formula.

$a_{100} = 6 + (100 - 1)5$ Substitute 100 for n, 6 for a, and 5 for d.

$a_{100} = 501$ Simplify.

The 100th term is 501.

B **What are the second and third terms of the arithmetic sequence 100, ■, ■, 82, . . .?**

The first term a is 100. The fourth term a_4 is 82. There are 3 common differences between 100 and 82.

$82 = 100 + 3d$ Add $3d$ to move from 100 to 82.

$-18 = 3d$ Solve for d.

$-6 = d$

The common difference is -6. The terms are 100, 94, 88, 82, ...

The second and third terms are 94 and 88.

Arithmetic Sequence

Analyzing Arithmetic Sequences

2. a. What is the 46th term of the arithmetic sequence that begins 3, 5, 7, . . . ?

b. What are the second and third terms of this arithmetic sequence?

80, ■, ■, 125, . . .

Arithmetic Sequence

Use the given table to write an explicit and a recursive rule for the sequence.

n	0	1	2	3	4	5
$f(n)$	2	5	8	11	14	17

First, check the differences of consecutive values of $f(n)$:

So, the explicit rule for the sequence is $f(n) = 2 + 3n$ for $0 \leq n \leq 5$.

The recursive rule is $f(0) = 2$

and $f(n) = f(n-1) + 3$ for $1 \leq n \leq 5$.

Arithmetic Sequence

Write an explicit and a recursive formula for each sequence.

1. 2, 4, 6, 8, 10, . . .

A recursive definition for this sequence has two parts:

$a_1 = a$ initial condition

$a_n = a_{n-1} + d$, for $n > 1$ recursive formula

An explicit definition for this sequence is a single formula:

$a_n = a + (n - 1)d$, for $n \geq 1$

2. 0, 6, 12, 18, 24, . .

1. $a_n = 2 + 2(n - 1)$; $a_n = a_{n-1} + 2$, $a_1 = 2$

2. $a_n = 0 + 6(n - 1)$; $a_n = a_{n-1} + 6$, $a_1 = 0$

Arithmetic Sequence

Write an explicit and a recursive formula for each sequence.

3. $-5, -4, -3, -2, -1, \ldots$

A recursive definition for this sequence has two parts:

$a_1 = a$ initial condition

$a_n = a_{n-1} + d$, for $n > 1$ recursive formula

An explicit definition for this sequence is a single formula:

$a_n = a + (n - 1)d$, for $n \geq 1$

4. $-4, -8, -12, -16, -20, \ldots$

3. $a_n = -5 + 1(n - 1)$; $a_n = a_{n-1} + 1$, $a_1 = -5$

4. $a_n = -4 - 4(n - 1)$; $a_n = a_{n-1} - 4$, $a_1 = -4$

Arithmetic Sequence

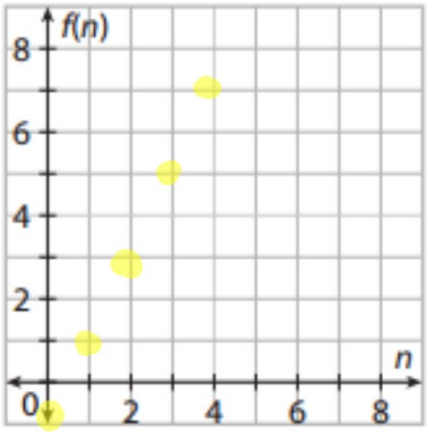
Graphing Arithmetic Sequences

Write the terms of the given arithmetic sequence and then graph the sequence.

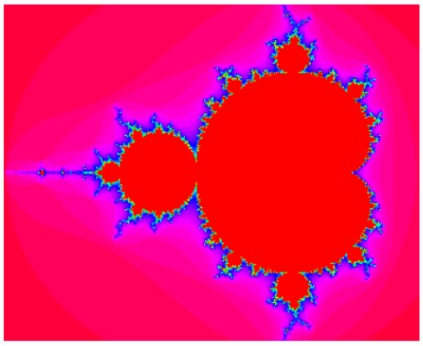
Ⓐ $f(n) = -1 + 2n$ for $0 \leq n \leq 4$

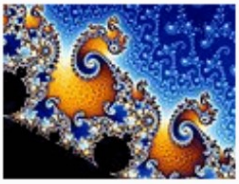
Make a table of values.

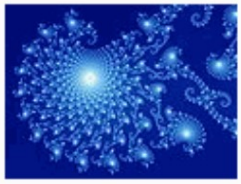
n	$f(n)$
0	$-1 + 2(0) = -1$
1	$-1 + 2(1) = 1$
2	$-1 + 2(2) = 3$
3	$-1 + 2(3) = 5$
4	$-1 + 2(4) = 7$

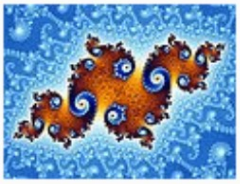


Recursive Sequence





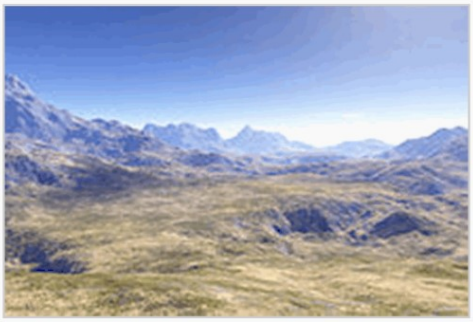




Julia and Mandelbrot Set

Recursive Fractals

Fractal images are actually being used nowadays to create special effects. Utilized in shows such as Star Trek and Star Wars, fractals are used to create landscapes that are otherwise impossible with conventional technology. On a related note, fractals are also used in creating some computer graphics.



An example of a fractal landscape

Fractal pastures

Fractal terrain

Arithmetic Sequence

Any Questions ?

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Classwork:

Worksheet 12.1

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